# The estimation of FCN (Free Core Nutation) period and quality factor from tidal gravity measurements at Józefosław, Poland

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## Abstract

In this paper we investigate in determination of free core nutation period (FCN) and quality factor from gravity measurements. This study is based on 3.5 year gravity records collected with use of LaCoste&Romberg Earth Tide gravimeter no. 26 located in Józefosław Observatory near Warsaw, Poland. We investigated in diurnal tidal gravity waves which are affected by fluid core resonance. From the enhancements of gravimetric factors and phases the eigenperiod of free core nutation was inferred to be equal to 430 sidereal days. This result is in good accordance with previous determination using gravimetric and VLBI techniques and confirms the discrepancy of the dynamic flattening of the outer core from its theoretical value for hydrostatic assumption. The value of quality factor (ca. 1300) is less than those obtained using VLBI which lead us to already reported conclusion that gravity measurements are more sensitive to site dependent phenomena (like atmospheric and indirect ocean tidal effects) than VLBI. In order to evaluate this phenomena correctly we investigated also in the importance of environmental correction for gravity measurements and their influence on determined FCN period.

## Tidal analysis results

Table: Tidal analysis results (diurnal band)

| | | | NC | | | | PC | | | | PC+OTLC | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $f$ [°/h] | $A_{th}$ $[\frac{nm}{s^2}]$ | $\delta$ | $m_\delta$ | $\varphi$ [°] | $m_\varphi$ [°] | $\delta$ | $m_\delta$ | $\varphi$ [°] | $m_\varphi$ [°] | $\delta$ | $\varphi$ |
| $Q_1$ | 13.399 | 57.7 | 1.1477 | 0.0017 | -0.0870 | 0.0840 | 1.1481 | 0.0008 | -0.0660 | 0.0410 | 1.1546 | 0.0085 |
| $O_1$ | 13.943 | 301.3 | 1.1504 | 0.0003 | 0.0720 | 0.0160 | 1.1504 | 0.0002 | 0.0930 | 0.0080 | 1.1541 | -0.0498 |
| $M_1$ | 14.497 | 23.7 | 1.1423 | 0.0042 | 0.1280 | 0.2120 | 1.1519 | 0.0021 | 0.1120 | 0.1040 | 1.1531 | -0.0803 |
| $\pi_1$ | 14.918 | 8.2 | 1.1436 | 0.0116 | 0.3290 | 0.5800 | 1.1632 | 0.0058 | -0.0420 | 0.2860 | 1.1628 | -0.1637 |
| $P_1$ | 14.959 | 140.2 | 1.1487 | 0.0007 | 0.2350 | 0.0340 | 1.1483 | 0.0003 | 0.1100 | 0.0170 | 1.1478 | 0.0068 |
| $S_1$ | 15.000 | 3.3 | 1.0835 | 0.0364 | -11.8850 | 1.9270 | 1.1767 | 0.0201 | -6.4250 | 0.9750 | 1.1763 | -6.5248 |
| $K_1$ | 15.041 | 423.6 | 1.1360 | 0.0002 | 0.1330 | 0.0100 | 1.1359 | 0.0001 | 0.0950 | 0.0050 | 1.1352 | -0.0001 |
| $\psi_1$ | 15.082 | 3.3 | 1.2799 | 0.0258 | 2.1780 | 1.1550 | 1.2682 | 0.0134 | 2.2900 | 0.6060 | 1.2673 | 2.2255 |
| $\varphi_1$ | 15.123 | 6.0 | 1.1746 | 0.0159 | -0.4040 | 0.7780 | 1.1704 | 0.0080 | -0.9310 | 0.3940 | 1.1694 | -0.9899 |
| $J_1$ | 15.585 | 23.7 | 1.1584 | 0.0029 | 0.0040 | 0.1450 | 1.1573 | 0.0014 | 0.0330 | 0.0720 | 1.1550 | 0.2339 |
| $OO_1$ | 16.139 | 13.0 | 1.1521 | 0.0039 | -0.3670 | 0.1960 | 1.1520 | 0.0020 | 0.0110 | 0.0980 | 1.1485 | 0.6568 |

## Tidal analysis results

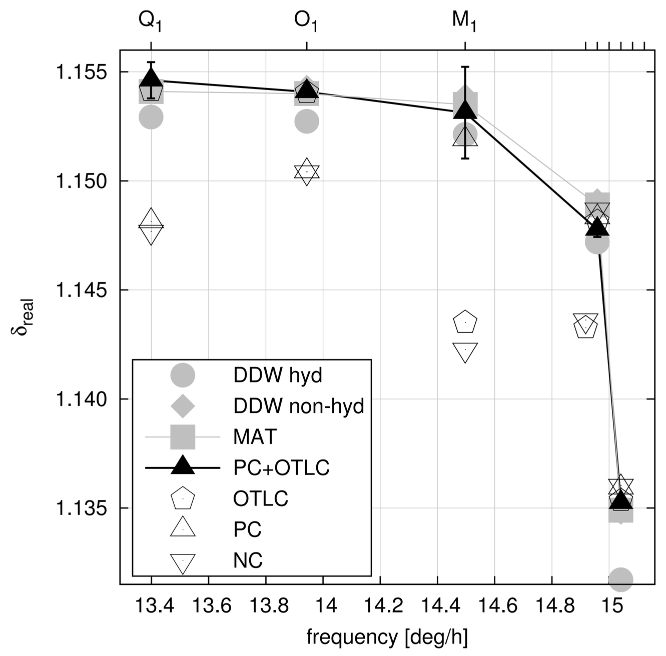


Figure: Comparison of gravimetric factors for diurnal tidal waves. The results when no correction (NC), pressure correction (PC) and ocean tidal loading correction (OTLC) was applied are shown along with theoretical models of Dehant, Defraigne, Wahr (DDW) and Matthews (MAT).

## Resonance curve

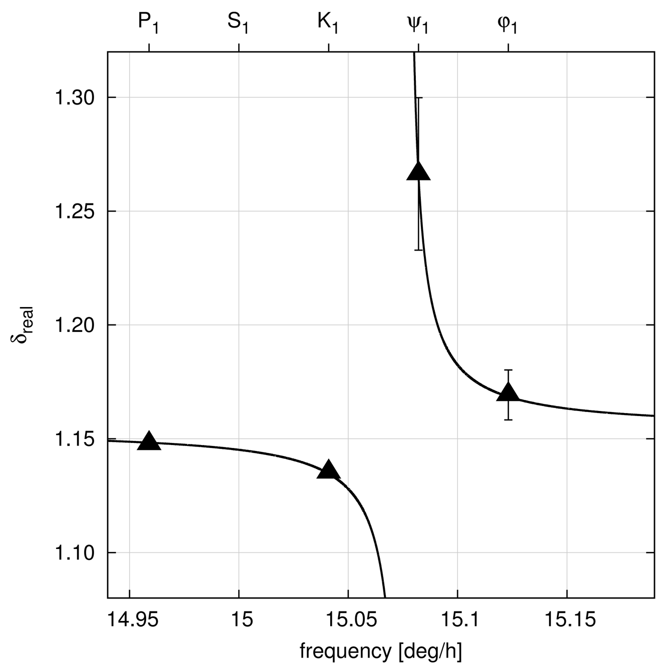


Figure: Resonance curve fitted to gravimetric factors

## Error estimation

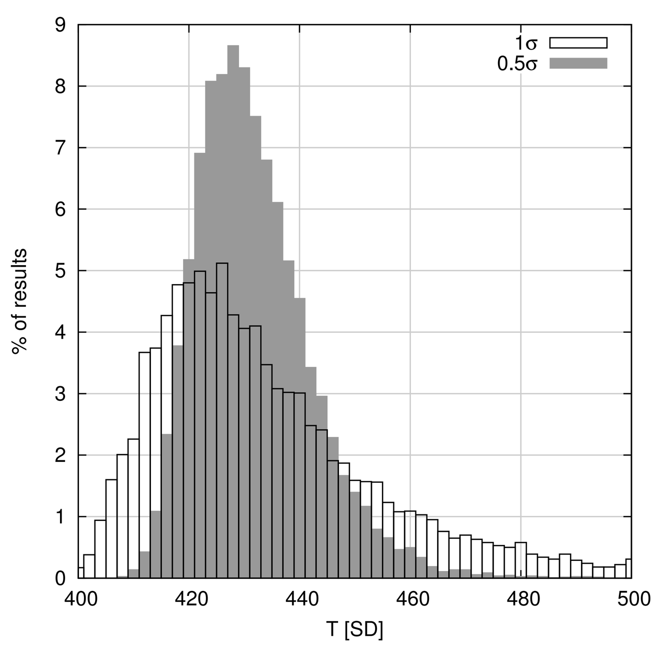


Figure: Monte Carlo simulation of results

## FCN parameters determination

Table: Comparison of FCN period determination using different strategies

| | $T$ [SD] | $A_r$ $[\cdot 10^4\ h \cdot deg^{-1}]$ | | $T$ [SD] | $A_r$ $[\cdot 10^4\ h \cdot deg^{-1}]$ | |
|---|---|---|---|---|---|---|
| Solution | $M_1,\ \pi_1,\ K_1, \psi_1,\ \varphi_1,\ J_1$ | | | $M_1,\ \pi_1,\ P_1,\ K_1, \psi_1,\ \varphi_1,\ J_1$ | | |
| NC | 408.3 (396.5 – 420.9) | 5.47 | (±0.20) | 412.0 (391.5 – 434.7) | 5.40 | (±0.32) |
| PC | 413.2 (402.0 – 425.0) | 5.45 | (±0.17) | 418.0 (397.1 – 441.3) | 5.37 | (±0.31) |
| OTLC | 423.2 (407.0 – 440.7) | 6.87 | (±0.30) | 421.4 (407.0 – 436.8) | 6.90 | (±0.27) |
| PC+OTLC | 430.2 (421.4 – 439.5) | 6.80 | (±0.15) | 426.0 (414.1 – 438.6) | 6.88 | (±0.21) |
| Solution | $K_1, \psi_1,\ \varphi_1$ | | | $P_1,\ K_1, \psi_1,\ \varphi_1$ | | |
| NC | 408.5 (401.7 – 415.5) | 5.47 | (±0.11) | 412.3 (385.8 – 442.6) | 5.39 | (±0.42) |
| PC | 413.2 (407.9 – 418.7) | 5.45 | (±0.08) | 418.1 (390.8 – 449.6) | 5.37 | (±0.41) |
| OTLC | 423.4 (419.9 – 426.9) | 6.86 | (±0.06) | 421.5 (412.9 – 430.5) | 6.90 | (±0.16) |
| PC+OTLC | 430.0 (429.8 – 430.2) | 6.80 | (±0.00) | 425.8 (410.6 – 442.2) | 6.88 | (±0.27) |

## Comparison with previous studies

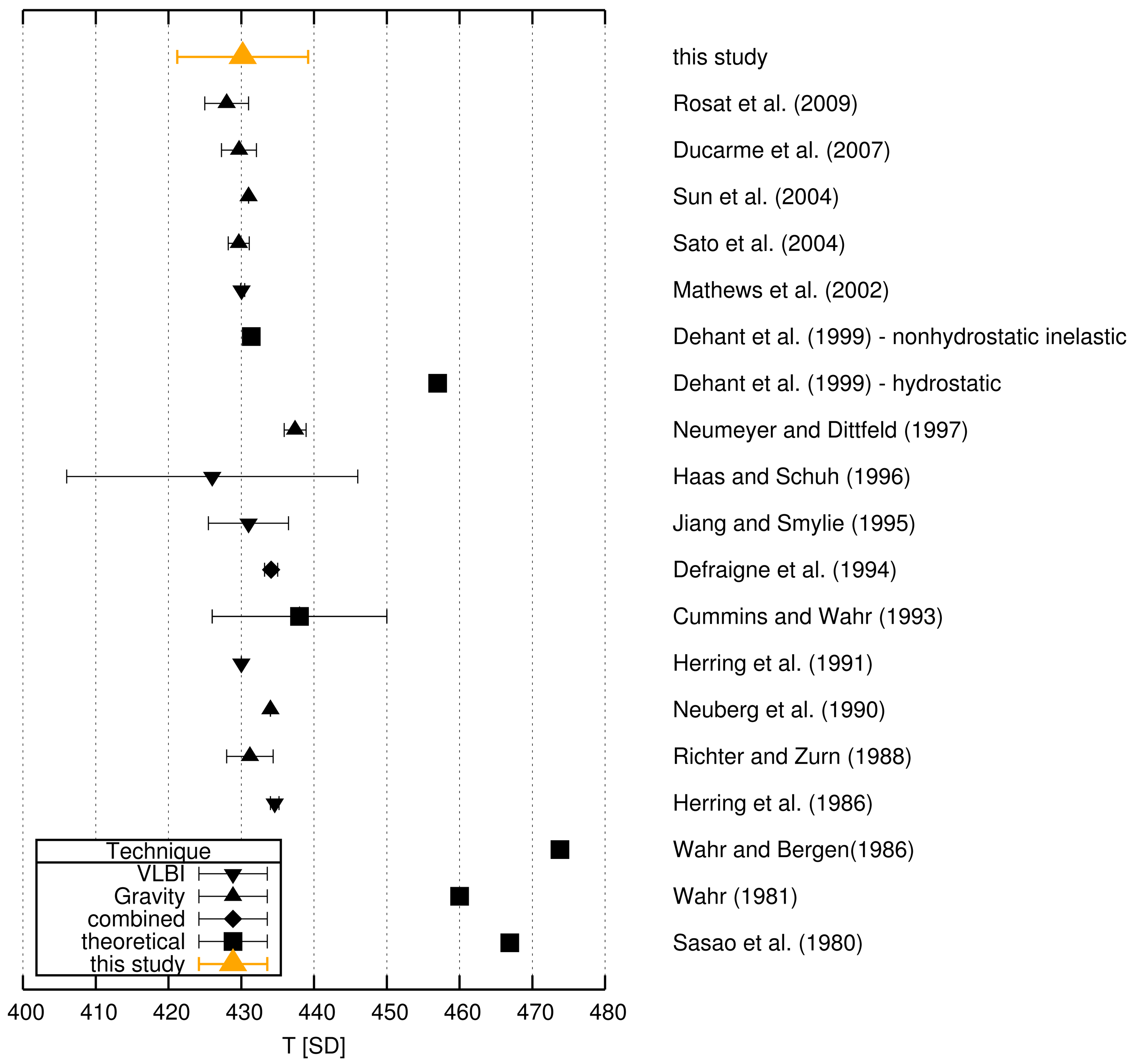


Figure: Comparison of FCN period determination from different studies

## Quality factor

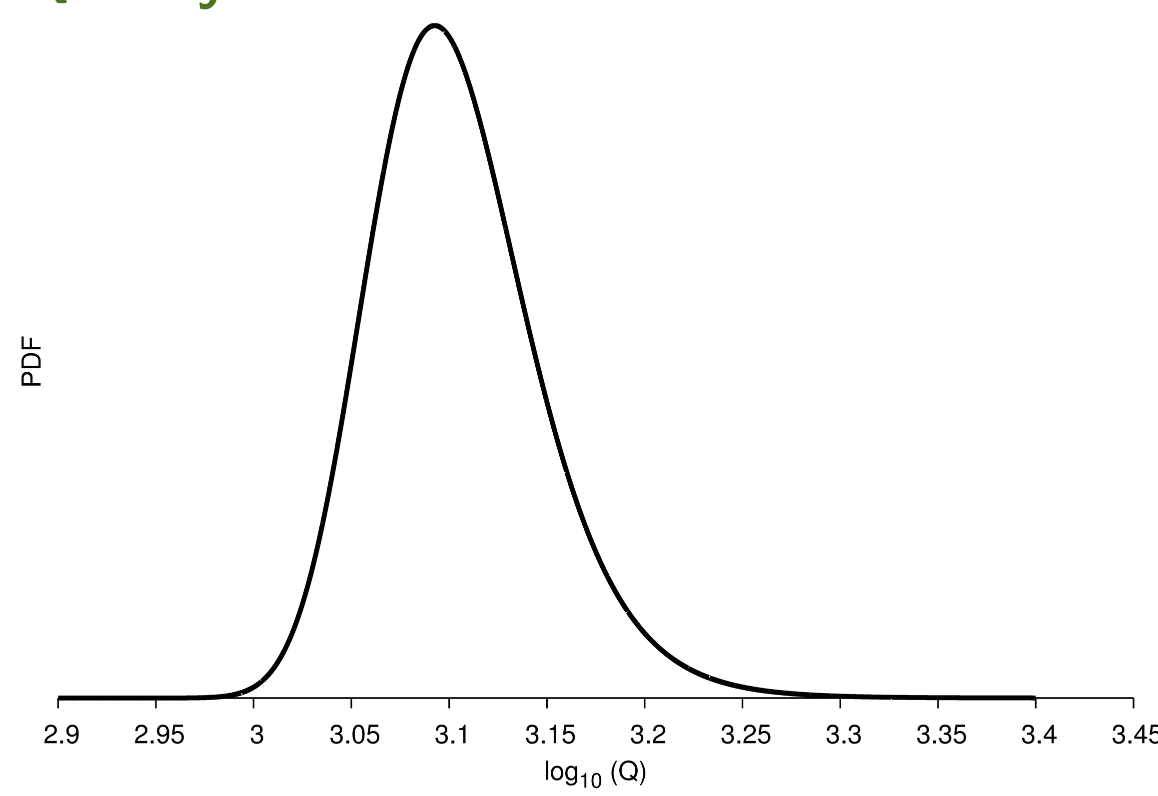


Figure: $Q$ estimation with Bayesian method

## Mathematical background

$$\tilde{\delta}(\sigma) = \delta_0 + \frac{\tilde{A}}{\sigma - \tilde{\sigma}_{NDFW}},$$

$$\tilde{\delta}(\sigma) - \tilde{\delta}(\sigma_{O_1}) = \frac{\tilde{A}}{\sigma - \tilde{\sigma}_{NDFW}} + \frac{\tilde{A}}{\sigma_{O_1} - \tilde{\sigma}_{NDFW}},$$

$$\sum_{j=1}^{n} p_j \left[ \tilde{\delta}(\sigma_j) - \tilde{\delta}(\sigma_{O_1}) - \frac{\tilde{A}}{\sigma_j - \tilde{\sigma}_{NDFW}} + \frac{\tilde{A}}{\sigma_{O_1} - \tilde{\sigma}_{NDFW}} \right]^2.$$

$$\tilde{\sigma}_{NDFW} = f(\sigma_1, \sigma_2, \sigma_3, \tilde{\delta}_1, \tilde{\delta}_2, \tilde{\delta}_3)$$

$$\frac{1}{T_{FCN}} = \frac{1}{T_{NDFW}} - 1;\ Q = \frac{\sigma^r_{NDFW}}{2\sigma^i_{NDFW}}$$

## Conclusions

- Despite of high background noise we confirmed the usefulness of spring gravimeters for capturing small amplitude signals.
- The estimated period is in very good agreement with previous studies while quality factor is at least one order smaller then those from VLBI or stacking multiple gravity measurements.
- Removing environmental signals is crucial in this studies.

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