Geomagnetic coordinates

excerpt from materials prepared by M. Kruczyk with some additional explanation

$$U(r,\theta,\lambda) = R\sum_{n=1}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n}\left[g_n^m \cos(m\lambda) + h_n^m \sin(m\lambda)\right]\cdot P_n^m\left(\cos\theta\right)$$

$$U(r,\theta,\lambda,t) = R\sum_{n=1}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n}\left[g_n^m(t)\cos(m\lambda) + h_n^m(t)\sin(m\lambda)\right]\cdot P_n^m\left(\cos\theta\right)$$

Formula describes the constant part of geomagnetic field (1st degree – dipole component plus residual field, 99% of observed magnetic field), where

- U – potential,
- r – radius,
- θ – colatitude ($90° - \varphi$),
- λ – longitude,
- n – degree and m – order
- g_n^m, h_n^m – gaussian coefficients of degree n and order m,
- P_n^m – associated Legendre functions of degree n and order m for argument $\cos\theta$,

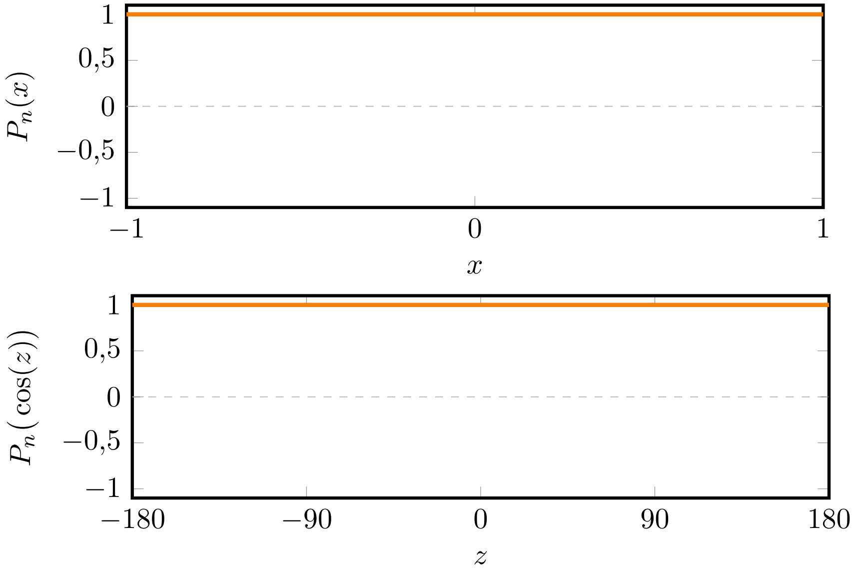
Legendre polynomials for $m = 0$

- $P_{n+1}(x) = \frac{2n+1}{n+1} x \cdot P_n(x) - \frac{n}{n+1} P_{n-1}(x)$

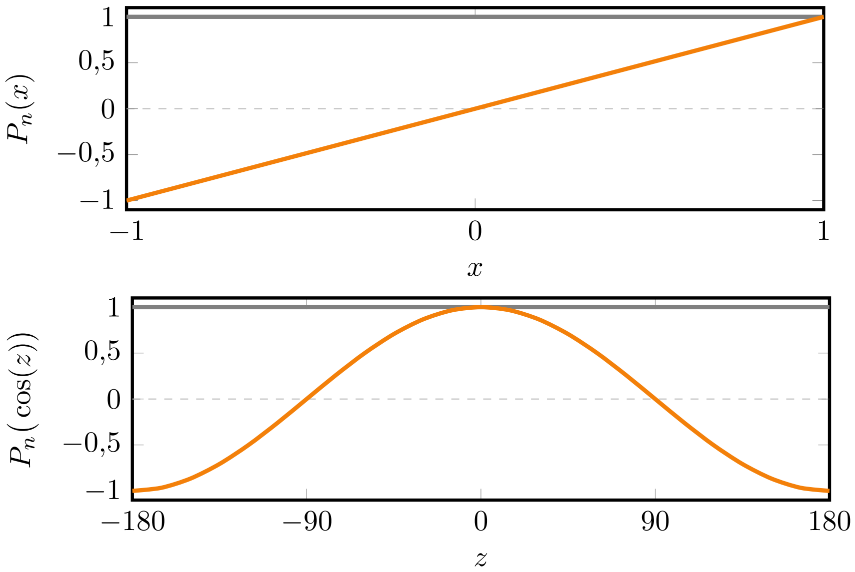
  recursive formula

- $P_n(x) = \frac{1}{2^n n!} \frac{\mathrm{d}^n}{dx^n} (x^2 - 1)^n$

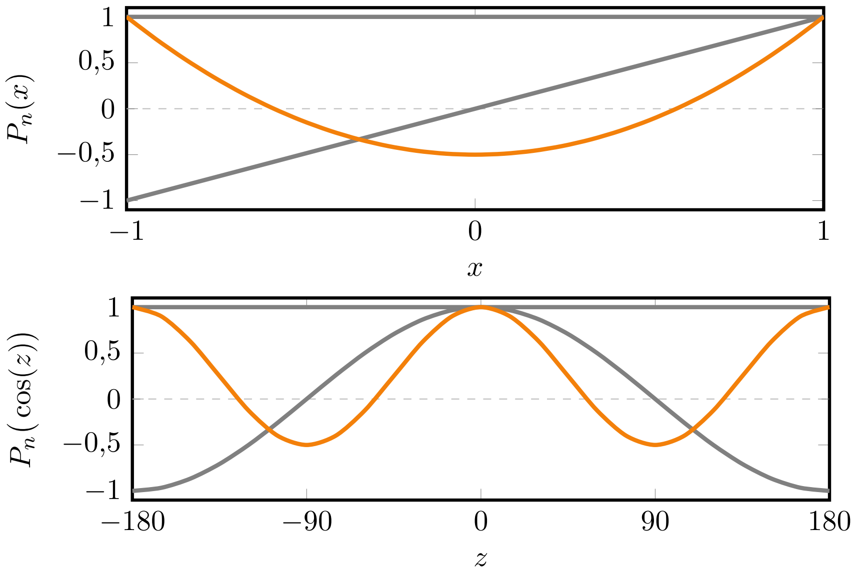
  Rodrigues formula



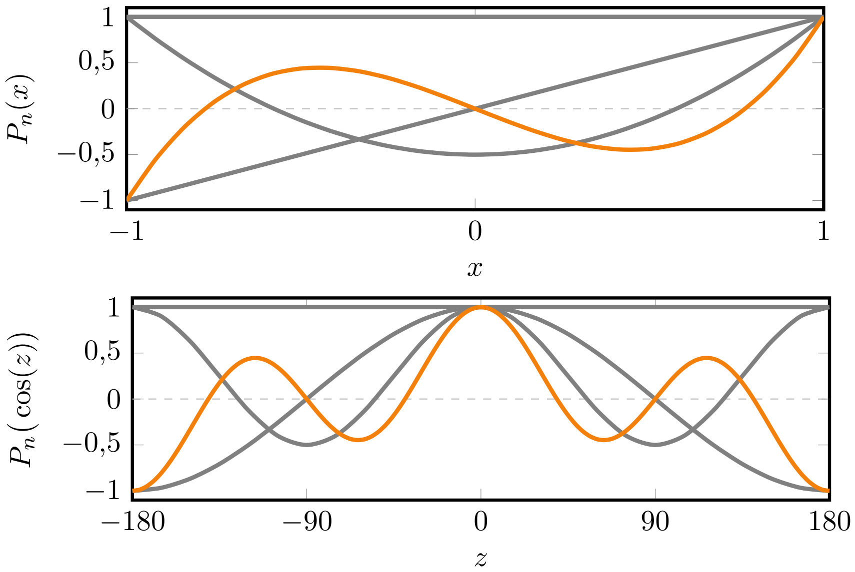
$$P_0(x) = 1$$



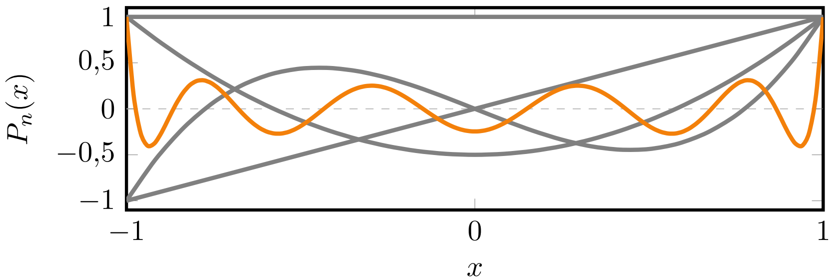
$$P_1(x) = x$$

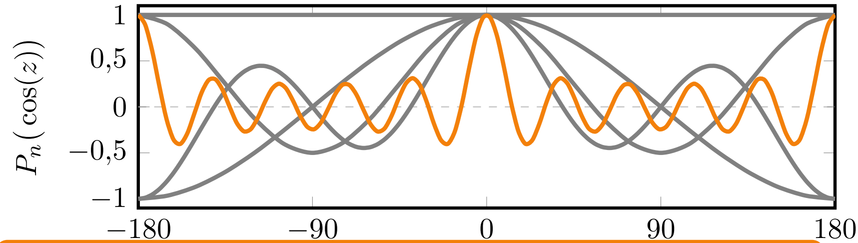


$$P_2(x) = 3/2 \cdot x^2 - 1/2$$



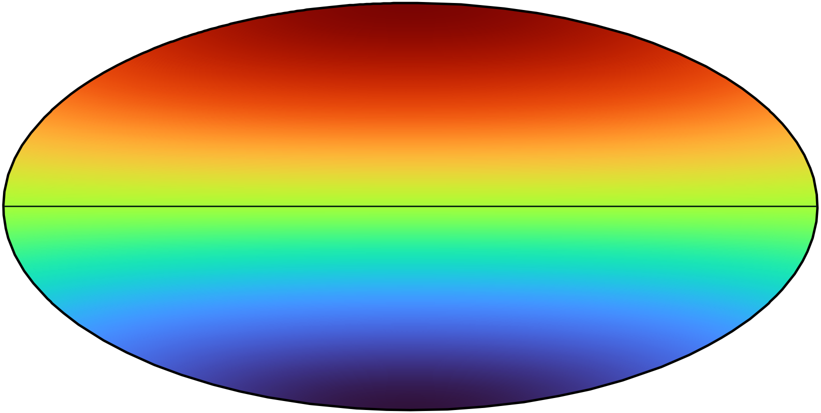
$$P_3(x) = 5/2 \cdot x^3 - 3/2 \cdot x$$



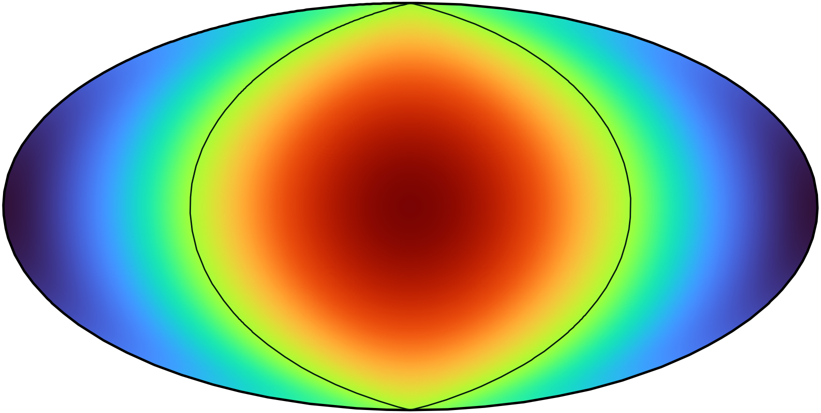


$$P_{10}(x) = 1/256 \cdot (46189 \cdot x^{10} - 109395 \cdot x^8 + 90090 \cdot x^6 - 30030 \cdot x^4 + 3465 \cdot x^2 - 63)$$

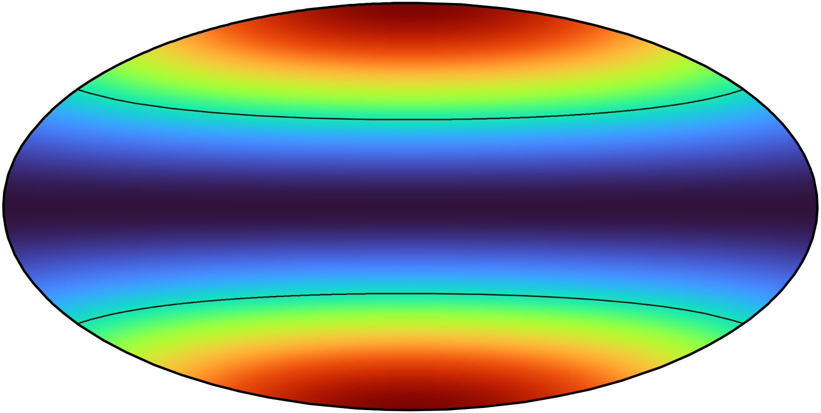
Spherical harmonics $n = 1$, $m = 0$



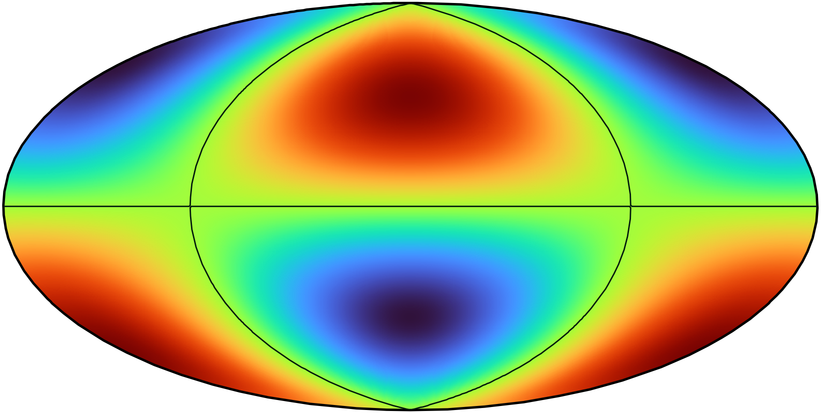
Spherical harmonics $n = 1$, $m = 1$



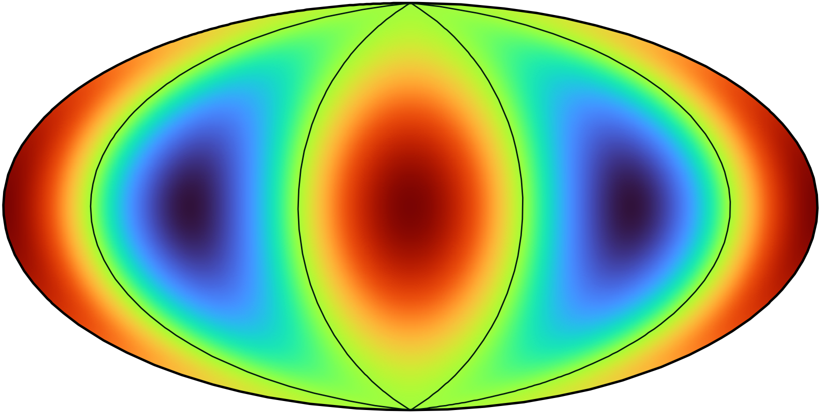
Spherical harmonics $n = 2$, $m = 0$



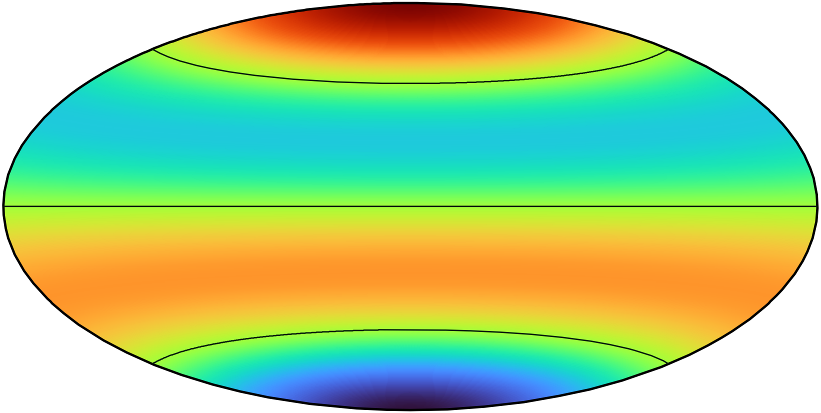
Spherical harmonics $n = 2$, $m = 1$



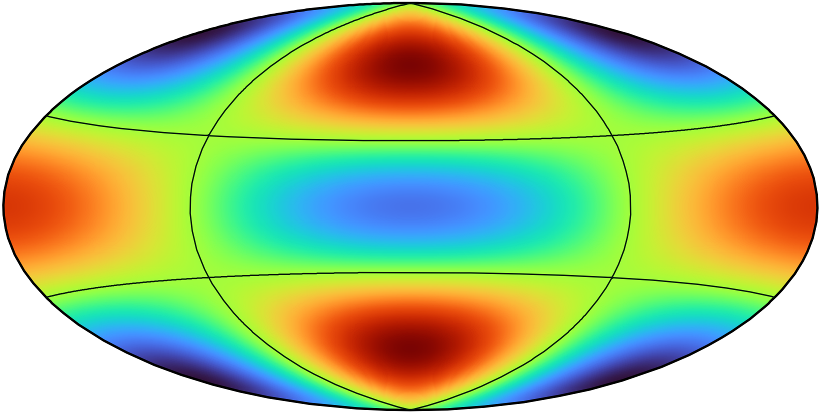
Spherical harmonics $n = 2$, $m = 2$



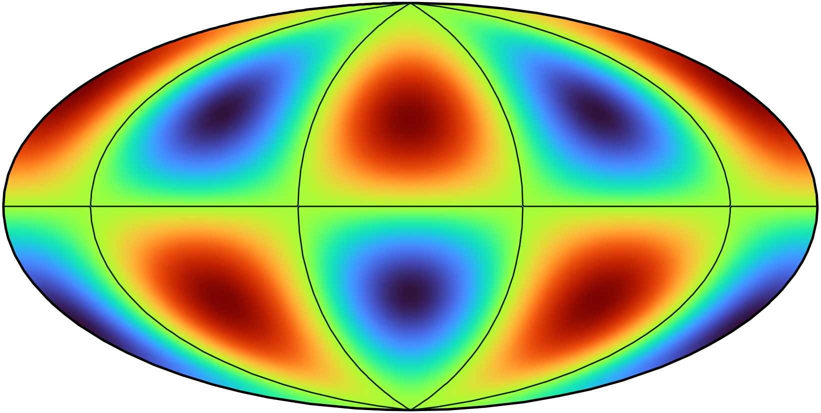
Spherical harmonics $n = 3$, $m = 0$



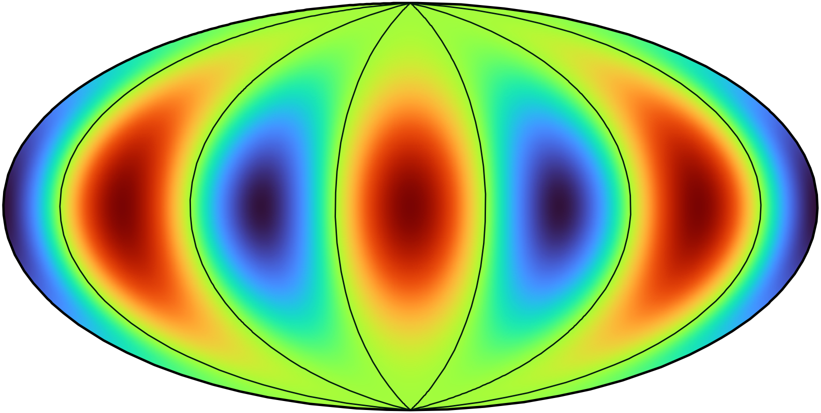
Spherical harmonics $n = 3$, $m = 1$



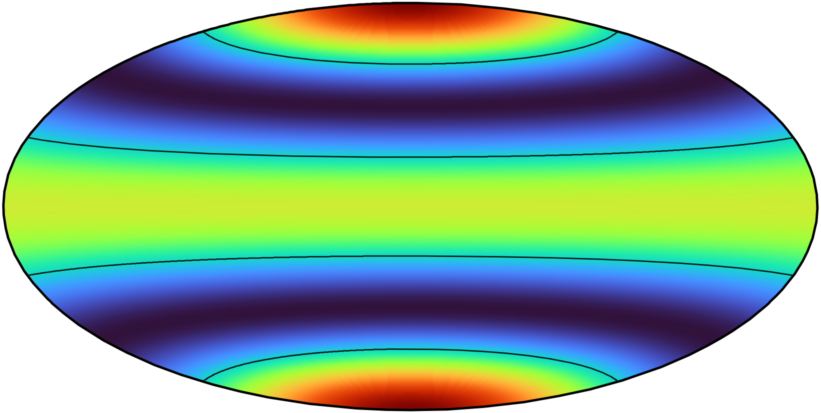
Spherical harmonics $n = 3$, $m = 2$



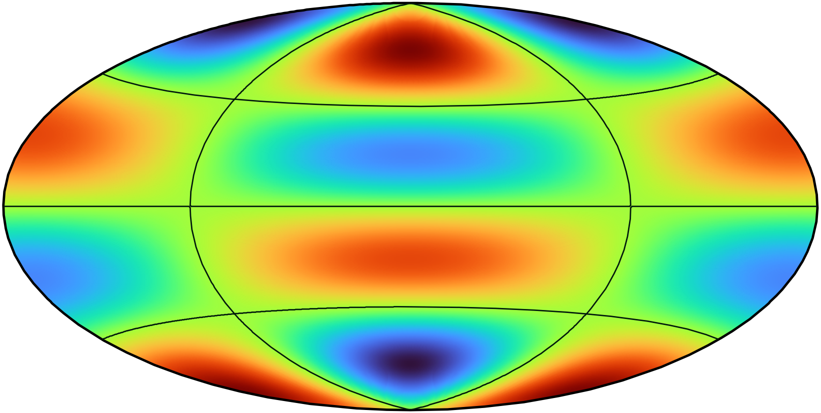
Spherical harmonics $n = 3$, $m = 3$



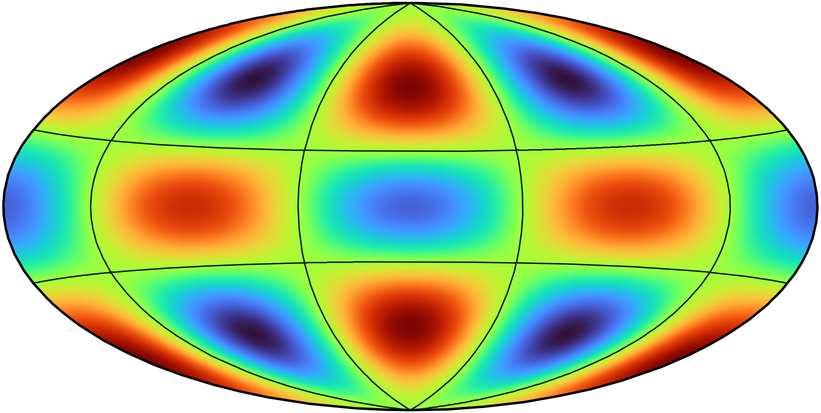
Spherical harmonics $n = 4$, $m = 0$



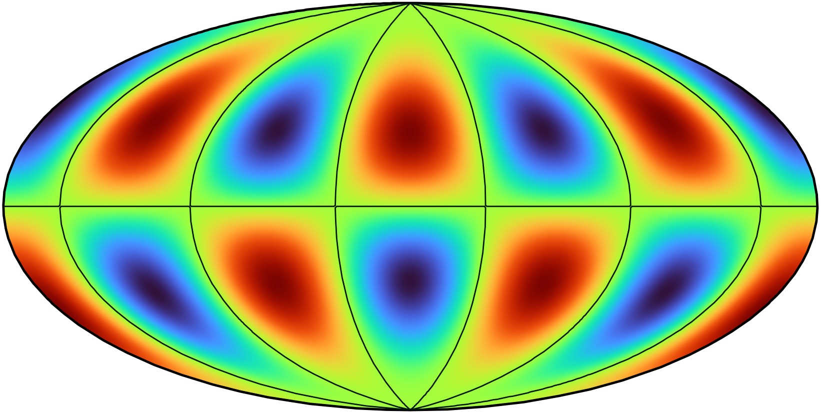
Spherical harmonics $n = 4$, $m = 1$



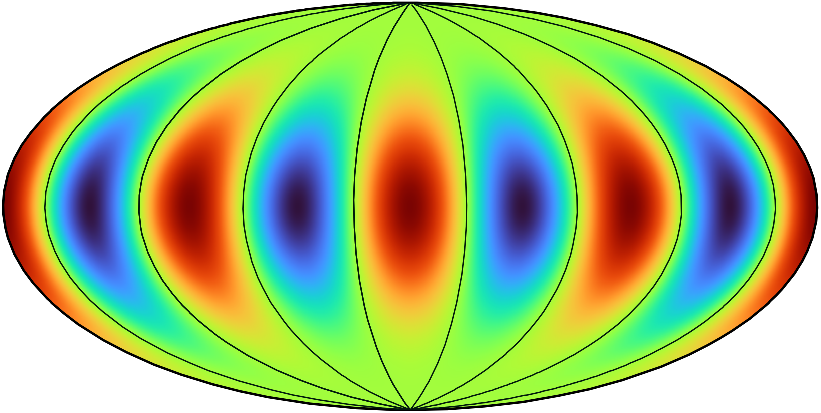
Spherical harmonics $n = 4$, $m = 2$



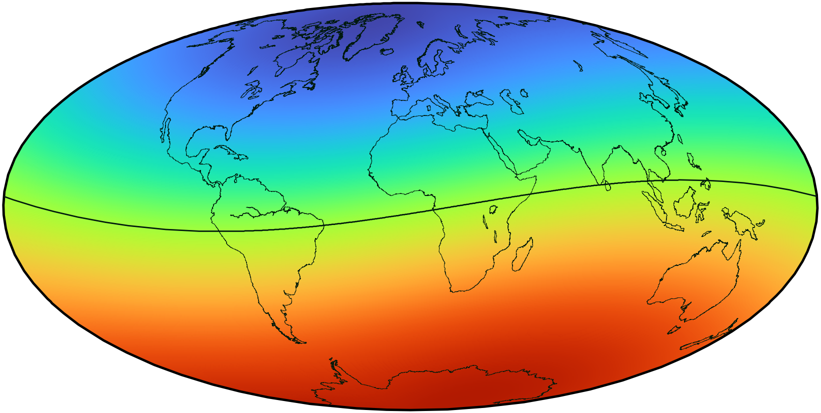
Spherical harmonics $n = 4$, $m = 3$



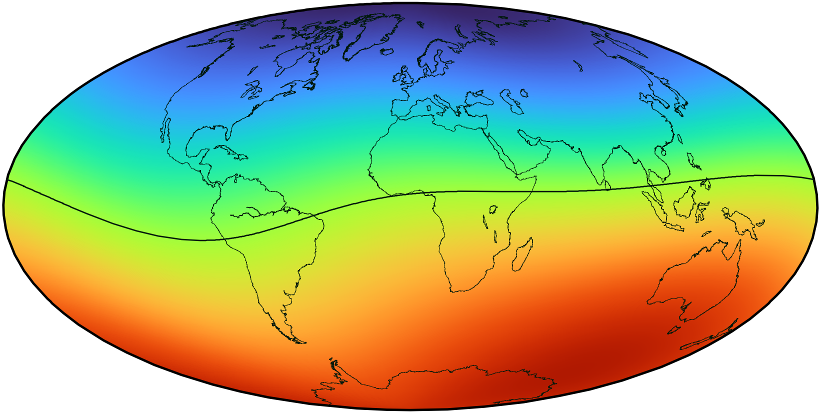
Spherical harmonics $n = 4$, $m = 4$



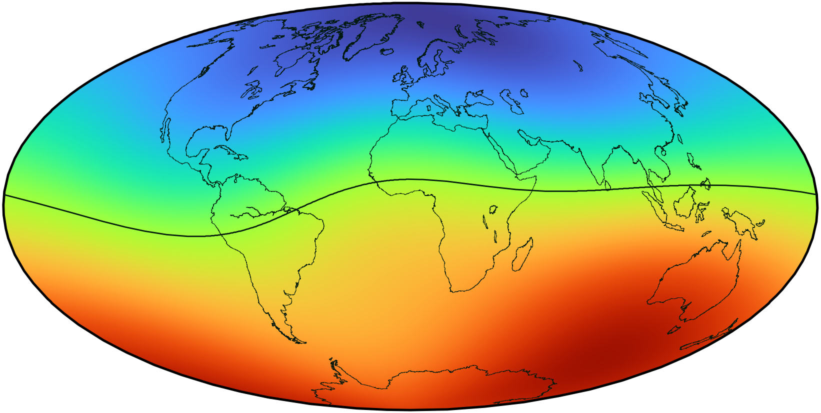
Spherical harmonics $n = 1$



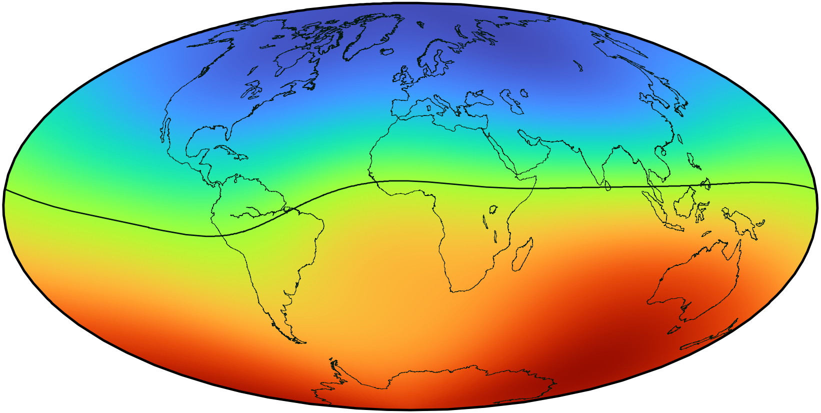
Spherical harmonics $n = 2$



Spherical harmonics $n = 3$



Spherical harmonics $n = 4$



Gaussian coefficients

`https://www.ngdc.noaa.gov/IAGA/vmod/coeffs/igrf13coeffs.txt`

$g_1^0 \quad g_1^1 \quad h_1^1$ 3

total: 3

Gaussian coefficients

`https://www.ngdc.noaa.gov/IAGA/vmod/coeffs/igrf13coeffs.txt`

$g_1^0 \quad g_1^1 \quad h_1^1$ 3

$g_2^0 \quad g_2^1 \quad h_2^1 \quad g_2^2 \quad h_2^2$ 5

total: 8

Gaussian coefficients

`https://www.ngdc.noaa.gov/IAGA/vmod/coeffs/igrf13coeffs.txt`

g_1^0	g_1^1	h_1^1					3
g_2^0	g_2^1	h_2^1	g_2^2	h_2^2			5
g_3^0	g_3^1	h_3^1	g_3^2	h_3^2	g_3^3	h_3^3	7

total: 15

Gaussian coefficients

`https://www.ngdc.noaa.gov/IAGA/vmod/coeffs/igrf13coeffs.txt`

g_1^0	g_1^1	h_1^1							3
g_2^0	g_2^1	h_2^1	g_2^2	h_2^2					5
g_3^0	g_3^1	h_3^1	g_3^2	h_3^2	g_3^3	h_3^3			7
g_4^0	g_4^1	h_4^1	g_4^2	h_4^2	g_4^3	h_4^3	g_4^4	h_4^4	9

total: 24

Gaussian coefficients

`https://www.ngdc.noaa.gov/IAGA/vmod/coeffs/igrf13coeffs.txt`

g_1^0	g_1^1	h_1^1							3
g_2^0	g_2^1	h_2^1	g_2^2	h_2^2					5
g_3^0	g_3^1	h_3^1	g_3^2	h_3^2	g_3^3	h_3^3			7
g_4^0	g_4^1	h_4^1	g_4^2	h_4^2	g_4^3	h_4^3	g_4^4	h_4^4	9

total: 24

Arithmetic sequence, so total numbers of coefficients is equal to:

$$(1 + 2 + 2 \cdot n + 1) \cdot \frac{n}{2} = n \cdot (n + 2)$$

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g_1^0	g_1^1	h_1^1							3
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g_3^0	g_3^1	h_3^1	g_3^2	h_3^2	g_3^3	h_3^3			7
g_4^0	g_4^1	h_4^1	g_4^2	h_4^2	g_4^3	h_4^3	g_4^4	h_4^4	9

total: 24
Arithmetic sequence, so total numbers of coefficients is equal to:

$$(1 + 2 + 2 \cdot n + 1) \cdot \frac{n}{2} = n \cdot (n + 2)$$

For IGRF13 (up to $n = 13$) there are 195 coefficients.

Excercise

Using model of IGRF13 (https://www.ngdc.noaa.gov/IAGA/vmod/coeffs/igrf13coeffs.txt) and assuming dipole model, compute magnetic intensity (3 components) and declination, and inclination for point of your choice (φ, λ).

Excercise

Using model of IGRF13 (`https://www.ngdc.noaa.gov/IAGA/vmod/coeffs/igrf13coeffs.txt`) and assuming dipole model, compute magnetic intensity (3 components) and declination, and inclination for point of your choice (φ, λ).

$$X = -g_1^0 \sin\theta + (g_1^1 \cos\lambda + h_1^1 \sin\lambda) \cos\theta$$

$$Y = g_1^1 \sin\lambda - h_1^1 \cos\lambda$$

$$Z = -2 \left[g_1^0 \cos\theta + (g_1^1 \cos\lambda + h_1^1 \sin\lambda) \sin\theta \right]$$

Excercise

Using model of IGRF13 (`https://www.ngdc.noaa.gov/IAGA/vmod/coeffs/igrf13coeffs.txt`) and assuming dipole model, compute magnetic intensity (3 components) and declination, and inclination for point of your choice (φ, λ).

$$
\begin{aligned}
X &= -g_1^0 \sin\theta + (g_1^1 \cos\lambda + h_1^1 \sin\lambda)\cos\theta \\
Y &= g_1^1 \sin\lambda - h_1^1 \cos\lambda \\
Z &= -2\left[g_1^0 \cos\theta + (g_1^1 \cos\lambda + h_1^1 \sin\lambda)\sin\theta\right]
\end{aligned}
$$

$$
\begin{aligned}
X &= -g_1^0 \cos\varphi + (g_1^1 \cos\lambda + h_1^1 \sin\lambda)\sin\varphi \\
Y &= g_1^1 \sin\lambda - h_1^1 \cos\lambda \\
Z &= -2\left[g_1^0 \sin\varphi + (g_1^1 \cos\lambda + h_1^1 \sin\lambda)\sin\theta\right]
\end{aligned}
$$

Total intensity:

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$$F = \sqrt{X^2 + Y^2 + Z^2}$$

Total intensity:

$$F = \sqrt{X^2 + Y^2 + Z^2}$$

Declination:

Total intensity:

$$F = \sqrt{X^2 + Y^2 + Z^2}$$

Declination:

$$D = \operatorname{atan} \frac{Y}{X}$$

Total intensity:

$$F = \sqrt{X^2 + Y^2 + Z^2}$$

Declination:

$$D = \operatorname{atan} \frac{Y}{X}$$

Inclination:

Total intensity:

$$F = \sqrt{X^2 + Y^2 + Z^2}$$

Declination:

$$D = \operatorname{atan} \frac{Y}{X}$$

Inclination:

$$I = \operatorname{atan} \frac{Z}{\sqrt{X^2 + Y^2}}$$